Modal Parameter Identification of A Continuous Beam Bridge by Using Grouped Response Measurements

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Abstract Experimental modal analysis of large civil structures such as bridges requires measurements of the bridge vibrations, which are generally expensive and arduous to obtain. In this study, modal parameter identification of a model bridge was performed using grouped response measurements which were obtained with a limited number of sensors. To illustrate the procedure, a continuous beam bridge was numerically modeled. Simulation responses of the model bridge were then obtained in groups where each of the group was considered to be acquired at a different time and due to a different excitation. White noise signals were added into the response signals to simulate real-life applications. The grouped simulation responses were obtained at different times, thereafter, each group responses were transformed into an equivalent single time interval. Transfer functions were evaluated between two consecutive groups by using the Fourier transform. Once the equivalent response data was obtained, modal parameters of the model bridge were calculated by a combination of the Natural Excitation Technique and Eigensystem Realization Algorithm method (NExT-ERA). Identification results of the NExT-ERA analysis were compared with the modal parameters of the numerical model.

1 Introduction

Experimental modal analysis of large civil structures such as bridges requires measurements of the bridge vibrations, which are generally arduous to obtain. Response measurements have been obtained with wired communication for years and today it is also possible to acquire such measurements with wireless communication by means of ongoing technological developments in wireless sensors. In the case of wired communication in long structures, environmental noise is very likely to enter the measured response signals. This means that acquired response measurements could not represent the actual structural response behavior when long signal cables are used. One way to reduce the noise in long cables is to use multiple data acquisition systems in the structure and have shorter distances to the sensors. In case wireless sensors are used, all the wireless sensors cannot communicate with a central data acquisition unit since wireless communication bandwidth is very limited. Thus, several data acquisition units need to be set up to acquire measurement data from distant sensors that are placed within a long or tall structure. As a result, the communication bandwidth of wireless sensors will stay in limited range [1].

The usage of multi-centered data acquisition units increases the cost for both wired and wireless communication. In addition to this, usage of a large amount of sensors requires large number of channels on data acquisition systems. This has a drawback on the maximum sampling rate of the data acquisition system. The maximum sampling rate of the measurement data decreases with the same proportion of increment in the number of channels used on data acquisition systems [2]. It can be said that the larger amount of sensors are used during measurements, the lower will be the sampling frequency. As a consequence of this, it will be difficult to identify higher modal frequencies of the structures using response measurements with a low sampling frequency.

All the aforementioned problems arise due to the usage of large numbers of sensors during measurements and therefore using smaller number of sensors for response measurements of structures can be considered as a solution for such problems. In this study, modal parameter identification of a model bridge was tried to be performed using grouped response measurements which were obtained with a limited number of sensors. To illustrate the procedure, a continuous beam bridge was numerically modeled within Matlab [3] environment. Simulation responses of the model bridge were then obtained in groups of which each was considered to be acquired at different times and due to different excitations. White noise signals were added into the grouped response signals to simulate real-life applications. After the grouped simulation responses were obtained at different times, each group responses were transformed into a single time interval. Finally, equivalent response data were employed in NExT-ERA to extract modal parameters of the model bridge. In this study, the aim was to be able to estimate the first 10 modes of the model bridge and compared them to the results of the eigenvalue analysis of the numerical model.

2 Finite Element Model

In order to implement the methodology, a two dimensional finite element model of a continuous beam bridge was set-up in a Matlab program. The model bridge has a total length of 180 meters which is composed of five spans of which each has a different length. The idea in assigning various lengths for the individual spans is to make it more complex for the system identification process. Then the total length is divided into 36 equal pieces of elements and each node in-between these elements has a vertical translational DOF and a rotational DOF. The structural stiffness matrix has a total of 74 DOFs, which consists of 37 vertical translational DOFs and 37 rotational DOFs. Axial deformations and second order effects were neglected in the analysis. The 6 support conditions and each DOF of the finite element model of the bridge used for implementation of the methodology are presented in Figure 1.

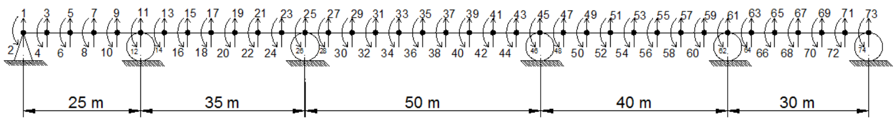


Figure 1. Finite element model of the bridge

The moment of inertia of the cross-section around the bending axis of the beams was considered to be 0.0731 m^4 in the analysis. Dimensions of the cross-section were determined so that the maximum vertical displacement of mid-span would be 6 cm. The damping matrix was constructed by using the mass-proportional damping formulation that is based on the Rayleigh Damping approach [4] and the modal damping ratio was considered to be 2% for all modes of the structure.

3 Implementation of the Methodology

The methodology was tested on the numerical bridge model that is described in the previous section. The 31 vertical unrestrained DOFs of the bridge model were aimed to be measured by using only a group of four sensors. The group of sensors was then shifted on the model in order to obtain the response measurements from all DOFs. For each shifting operation, the location of one sensor in each group was unchanged and this sensor was considered to be a reference sensor between the two consecutive groups. In Figure 2, the placement of the sensors in the first three groups is represented on the model bridge.

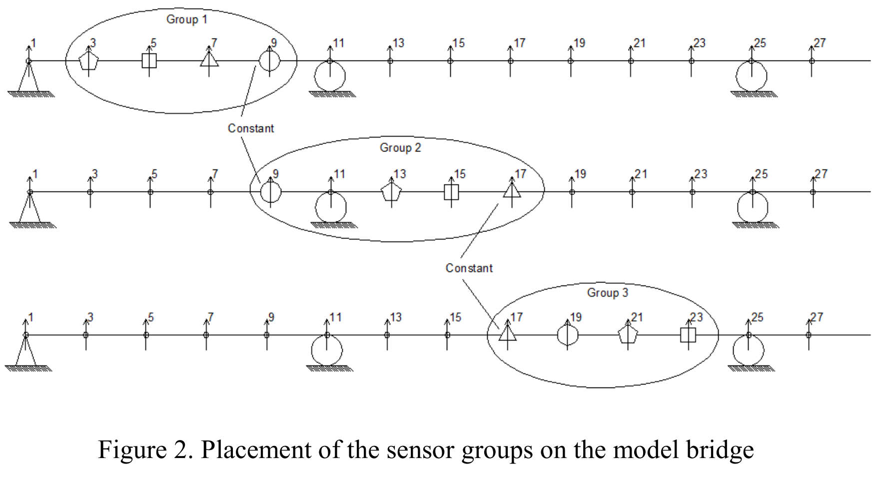


Figure 2. Placement of the sensor groups on the model bridge

As it is clearly shown in the figure, the rightmost sensor in each group was assigned as the reference sensor and thus a reference signal was obtained between two consecutive groups. According to the figure, the reference signal between group 1 and group 2 was obtained by the circle-shaped sensor on the 9^{th} DOF. Similarly, the reference signal between group 2 and group 3 was obtained by the triangle-shaped sensor on the 17^{th} DOF. All the other groups are placed on the model in the same sense.

As a result, ambient vibration responses of each DOF of the model are obtained by a total of 10 group measurements by using four sensors, only.

3.1 Generation of Response Data for Group Measurements

Since there is a total of 37 vertical translational DOFs in the numerical model, it is impossible to obtain response measurements from all DOFs at the same time by using 4 sensors only. Therefore, response measurements in each group should be considered to be obtained at different time intervals. In order to have ambient vibration responses at different time intervals for each group, the bridge model was excited by 10 different generic signals. It should be noted here that according to Caicedo [5,6], it is required to use long durational response measurement records for system identification using NExT-ERA in order to identify low mode frequency behavior of structures such as bridges. For this purpose, 10 different white noise excitation signals with a duration of 30 minutes are generated. Even though there is a need for long measurement records in NExT-ERA to obtain low frequency behavior, there is no need to acquire the data with a high sampling rate. Here, the generated white noise excitation signals have a sampling frequency of 200 Hz. These white noise signals are different along the time line, but they are stationary signals which have a constant mean and standard variation – statistically they are identical. In order to obtain non-stationary excitation signals, 10 different ground motion records with reduced amplitudes were included into the white noise signals. Since none of the recorded ground motions have a duration of 30 minutes, each ground motion signal is added to the end of itself until the total duration becomes 30 minutes.

Once, 10 different excitation signals were generated, the numerical model was excited by each of the generated excitation signals so that 10 different simulations were performed by using the Newmark β method with the constant average acceleration approach [4]. To perform the simulations, state-space model of the structure was constructed for each group one by one. After the acceleration response data were obtained for each group, random noise is added to the acceleration response of each DOF in order to imitate measurement noise effects as encountered in real life. The noise was set for each acceleration response to have a root mean square (RMS) of 10% of the RMS of the response itself. Since the amount of noise in the response data adversely affects identification of the modal parameters, noise level in each response signal had to be reduced. To this end, each group of acceleration signals were filtered by using a Kalman filter to increase the signal-to-noise ratio of the generated noisy acceleration signals. Kalman filter was designed for each group measurement. The Kalman filters were designed based on the numerical bridge model with acceleration readings of the corresponding group, only. The filter characteristics are yet to be investigated for errors in the bridge model.

As a result, acceleration response of each DOF with reduced noise level was obtained as if the response data in a simulation were measured at a different time with respect to another simulation. Acceleration response data were obtained with a duration of 30 minutes and with a sampling frequency of 200 Hz like the excitation signals. However, in order to focus on the identification of lower modes of the numerical model using NExT-ERA method, acceleration response data were down-sampled to a lower sampling frequency. In this study, it was aspired to determine the first 10 modes of the model bridge. According to the eigenvalue analysis of the numerical model, the resonant frequency of the 10^{th} mode has a value of 23.58 Hz. Therefore, in order to successfully identify the first 10 modes of the model, the acceleration response data were downsampled to 50 Hz. It was expected to be able to identify the first 10 modes of the model, since the down-sampled data have a Nyquist frequency of 25 Hz which covers the resonant frequencies up to the 10^{th} mode. The response signals were downsampled by first applying a low-pass anti-aliasing filter so that the modal responses with higher frequencies would be completely removed from the signals. Consequently, all the acceleration response measurements in the groups were obtained with 10 different simulations in accordance with the sensor placement represented in Figure 2.

3.2 Transformation of Grouped Response Measurements into the Equivalent Response Data

In order to obtain correlations between the response measurements obtained from different DOFs of a structure, all the response signals which will be used to obtain cross-correlation functions should be measured at the same time to be employed in NExT. Therefore, response measurements in groups, of which each is obtained in a different time interval, should be transformed into an equivalent response time frame to be used in NExT. As discussed in the previous section, the location of one sensor was kept fixed between two consecutive groups while the remaining sensors were being shifted towards the next group measurement. Thus, there are actually two different response measurements for a reference sensor between two consecutive groups. In order to perform the transformation of signals, the reference signals between the groups were employed. By using the transfer function between two response measurements on a reference sensor, response measurements in a group can be transformed into their equivalents in another group. In order to obtain transfer functions between two consecutive groups, all the acceleration response signals in the time domain were transformed into the frequency domain by applying Fast Fourier Transforms (FFT). In this study, the target was to estimate the response data which are equivalent to the response data obtained in the 1st simulation. Therefore, all grouped measurements were transformed into their equivalent responses with the 1st simulation. It should be noted here that since the response measurements in group 1 are the portion of the response measurements in the 1st simulation, the transformed response measurements of the remaining groups were expected to become as if measured at the same time with the response measurements in group 1.

In order to formulate the transformation procedure according to the sensor configuration shown in Figure 2, let the FFTs of the acceleration response signals obtained from 3rd, 5th, 7th and 9th DOF in group 1 be $u_3^1(\omega)$, $u_5^1(\omega)$, $u_7^1(\omega)$ and $u_9^1(\omega)$, respectively. Also, let the FFTs of the acceleration response signals obtained from 9th, 13th, 15th and 17th DOF in group 2 be $u_9^2(\omega)$, $u_{13}^2(\omega)$, $u_{15}^2(\omega)$ and $u_{17}^2(\omega)$, respectively. The transformation between group 1 and group 2 is derived by the function expressed in equation 1.

$$\alpha_{21}(\omega) = \frac{u_9^{\ 1}(\omega)}{u_9^{\ 2}(\omega)} \tag{1}$$

In the equation, the subscript of $u(\omega)$ represents the DOF number from which $u(\omega)$ was obtained and superscript of $u(\omega)$ represents the group number to which $u(\omega)$ belongs to. $\alpha_{21}(\omega)$ is the transformation coefficient to transform the response measurements of group 2 into the equivalent response measurements of group 1.
Then, each response measurement in group 2 was multiplied by $\alpha_{21}(\omega)$ to obtain the response measurements of group 2 which are equivalents of the response measurements in group 1 as shown in the expressions:

$$u_{13}^{\ 1}(\omega) = \alpha_{21}(\omega) \bullet u_{13}^{\ 2}(\omega) \tag{2}$$

$$u_{15}^{\ 1}(\omega) = \alpha_{21}(\omega) \bullet u_{15}^{\ 2}(\omega) \tag{3}$$

$$u_{17}^{\ 1}(\omega) = \alpha_{21}(\omega) \bullet u_{17}^{\ 2}(\omega) \tag{4}$$

According to the above expressions, $u_{13}^1(\omega)$, $u_{15}^1(\omega)$ and $u_{17}^1(\omega)$ represent the response measurements of group 2 which are transformed into the equivalents in group 1.

By using the similar procedure, $\alpha_{31}(\omega)$ is obtained which is the transformation function used to transform the response measurements of group 3 into the equivalents in group 1 and it is calculated using the following expression;

$$\alpha_{31}(\omega) = \alpha_{32}(\omega) \bullet \alpha_{21}(\omega) \tag{5}$$

where $\alpha_{31}(\omega)$ is the transformation coefficient which was used to transform response measurements from group 3 into the equivalents in group 2 and it is defined as

$$\alpha_{32}(\omega) = \frac{u_{17}^{\ 2}(\omega)}{u_{17}^{\ 3}(\omega)} \tag{6}$$

As it is clearly understood from the expression (5), while performing the transformation, the response measurements in group 3 were firstly transformed into their equivalents in group 2 and then these transformed equivalents were transformed into their equivalents in group 1. Calculating the transformation function $\alpha_{31}(\omega)$, each response measurement in group 3 was multiplied by $\alpha_{31}(\omega)$ to obtain the response measurements of group 3 which are equivalents of the response measurements in group 1 as shown in the expressions;

$$u_{19}^{1}(\omega) = \alpha_{31}(\omega) \bullet u_{19}^{3}(\omega) \quad (7)$$

$$u_{21}^{1}(\omega) = \alpha_{31}(\omega) \bullet u_{21}^{3}(\omega) \quad (8)$$

$$u_{23}^{1}(\omega) = \alpha_{31}(\omega) \bullet u_{23}^{3}(\omega) \quad (9)$$

In the above expressions, $u_{19}^{1}(\omega)$, $u_{21}^{1}(\omega)$ and $u_{23}^{1}(\omega)$ represent the response measurements of group 3 which are transformed into the equivalents in group 1.

Using the same procedures provided above, response measurements in all groups were transformed into their equivalents in group 1. Since the transformed results obtained by the above procedures are in the frequency domain, they were transformed into the time domain by the Inverse Fast Fourier transform (IFFT) in order to be employed in NExT. As a result, the equivalent time-domain response data of the bridge model which were expected to be equivalent with the response data in the 1st simulation were obtained using the grouped response measurements. Thus, the equivalent response data obtained by the transformation process were able to be employed in NExT-ERA in order to estimate modal parameters of the bridge model.

4 Identification Results and Conclusion

After the transformation, the equivalent response data were employed in NExT-ERA to identify the modal parameters of the model. The equivalent measurement of each DOF was used as a reference channel one by one also changing the model order of the system during the identification process and so many different identification processes were performed in order to be able to separate the true resonant frequencies of the model from computational frequencies [7]. So as to visually inspect consistency of the true modes when different model orders were used with different reference channels, stabilization diagrams were also plotted within Nyquist frequency range for each identification process.

True modes of the numerical model were expected to be consistent in almost all the stabilization diagrams. Figure 3 demonstrates two of the plotted stabilization diagrams in the identification process. As seen from the diagrams in the figure, specific frequencies show a high consistency when a different reference channel was selected for the identification process and therefore the frequencies which are consistent in almost all the stabilization diagrams were selected as the true modal frequencies of the numerical model.

To be able to verify the modal parameters obtained by using the equivalent response data in NExT-ERA, modal parameters (modal frequencies, modal damping ratios and mode shapes) of the numerical model were calculated by an eigenvalue analysis and were considered as the true modal parameters of the bridge model.

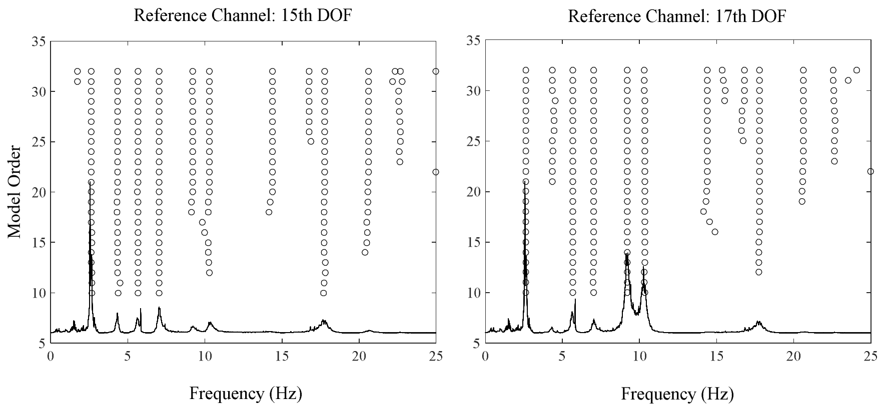


Figure 3. Stabilization Diagrams within the Nyquist Frequency Range

In Table 1, the modal frequencies and modal damping ratios which were identified by using the equivalent response data in NExT-ERA were compared with the actual modal frequencies and actual damping ratios of the first 10 modes of the model. According to the results of the modal frequency identification represented in the table, the first 10 modal frequencies of the model have been successfully identified with a maximum error of 2.53% by using the equivalent response data obtained by the transformation process.

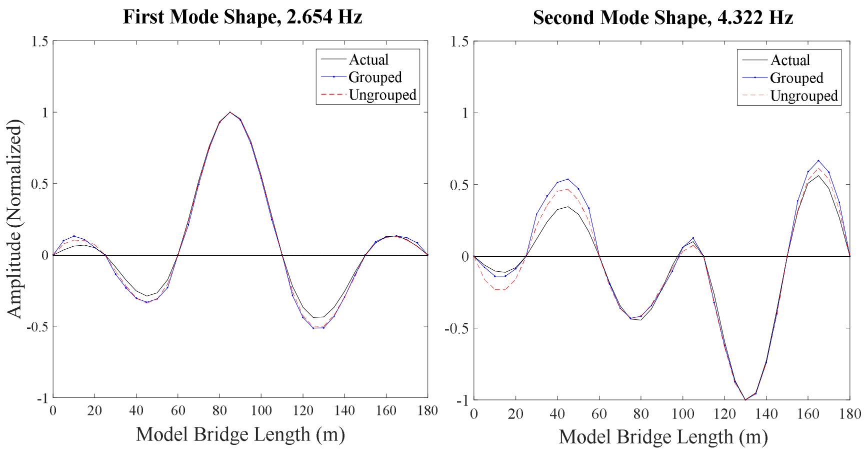
Table 1. Comparison of modal frequencies and damping ratios of the model bridge

# Mode	Actual Freq. (Hz)	Identified Freq. (Hz)	Error (%)	Actual Modal Damping Ratios (%)	Identified Modal Damping Ratios (%)	Error (%)
1	2.654	2.660	0.23	2	2.03	1.5
2	4.322	4.337	0.35		1.97	1.5
3	5.644	5.652	0.14		2.64	32
4	7.052	7.047	0.07		2.51	25.5
5	9.256	9.201	0.59		3.09	54.5
6	10.425	10.428	0.03		2.82	41
7	14.646	15.01	2.53		2.09	4.5
8	18.251	18.078	0.95		2.02	1
9	21.375	21.118	1.20		2.90	45
10	23.583	23.576	0.03		2.46	23

According to the results of the modal damping ratio identification presented in the table, although some identification results have minor errors within the acceptable limits, many of the results have major errors above the acceptable limits such as the identification result of the modal damping ratio of the fifth mode which has a maximum error of 54.5%. A large error in modal damping ratio estimation such as the case in this study is a well-known fact among system identification researchers. Nayeri et al. [8] explained this problem as “modal damping estimation is always crude and not as accurate as the modal frequency estimation” in the system identification methods including NExT-ERA. In addition, Moaveni [9] observed that “the natural frequencies using different methods are reasonably consistent while the identified damping ratios exhibit much larger variability across system identification methods.”

Nonetheless, for the validation purpose and to examine whether the major errors in identification of the modal damping ratios are caused by the transformed responses or not, the response measurements of the 1st simulation which are ungrouped measurements were directly employed in NExT-ERA and modal parameters of the numerical model were also identified in this way. The identification results have demonstrated that using the direct response measurements also results in similar major errors in identification of the modal damping. Thus, this result has validated that major errors in identified modal damping ratios are independent of using grouped measurements in the identification process.

The first 5 mode shapes of the numerical model were successfully identified in NExT-ERA by using the equivalent response measurements. The identified mode shapes were verified by comparing with the actual mode shapes obtained from eigenvalue analysis of the numerical model. They were also compared with the mode shapes identified by using the direct response measurements of the 1st simulation in NExT-ERA. Comparisons of the first 5 mode shapes of the bridge model are represented in Figure 4.



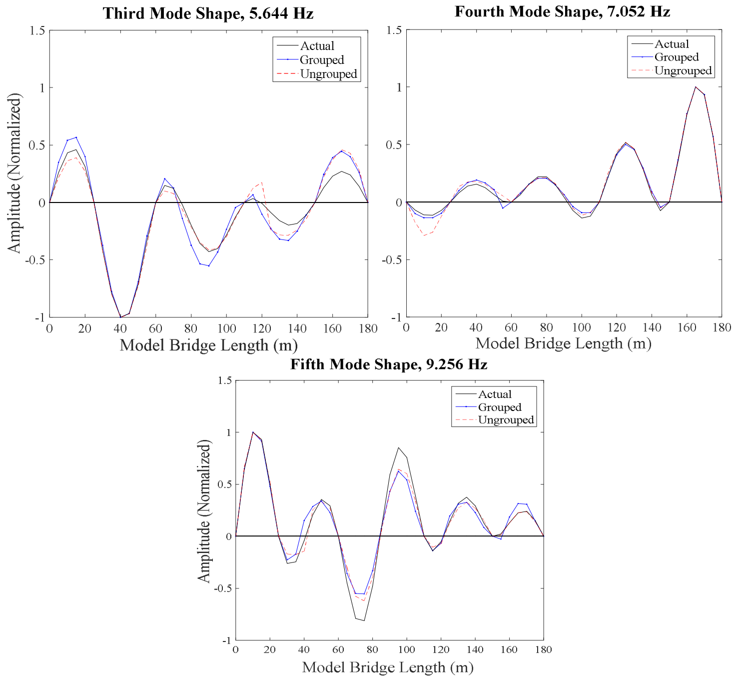


Figure 4. Comparison of the first 5 mode shapes of the numerical model

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