EVALUATION OF STEEL BRIDGE TRUSS SYSTEMS BY CONVERSION TO EQUIVALENT BOX GIRDERS

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Abstract Designing a three dimensional (3-D) truss system as a combination of two dimensional plane trusses ignores the torsional character of the system and often results in larger than necessary truss member sizes. In this study, the possibility and accurateness of conversion of 3-D truss systems into equivalent closed box sections for the examination of torsional behavior of 3-D truss systems were investigated. Twelve 3-D steel truss models and twelve equivalent steel box girder models were studied under five different loading conditions. The equivalent box girder models were obtained by the analytical conversion of each side (2-D trusses) of a 3-D truss structure to equivalent plates by strain energy method. Two types of conversions, named as partial and full conversion, were investigated. Finite element analysis and the classical mechanics method were tools for the analysis. Finite element analysis was used to obtain the deflection values of the truss systems and of the converted box girders. Results of analyzing the equivalent box girders by the classical method to investigate the applicability of the conversion method provided estimation of deflections of the 3-D truss systems.

1 Introduction

In current practice, steel bridge truss systems are designed as 2-D members without taking into account the torsional capacity of actual three dimensional (3-D) structure. This condition results in over designed structures with larger than necessary truss members. If the concept of equivalent plate of open steel box girders can be applied innovatively to truss structures, over design can be avoided and 3-D steel truss systems can be a better alternative to other types.

Kollbrunner - Basler [7] and Roik - Carl - Lindner [10] conducted analytical work to convert open steel box sections with bracing to closed steel box sections by replacing the bracing with a fictitious equivalent wall element of constant thickness (t_{eq}) throughout the length of the box. McDonald [9] and Heins, Snyder

& Blank [6] conducted experimental studies on open steel box sections. Conceptually, each side of a 3-D truss structure of prismatic shape can be converted to an equivalent plate, forming an equivalent box girder. In this study, the possibility and accurateness of the utilization of this concept of a fictitious box girder were investigated.

The specific objectives of this study were: 1) to develop a procedure of converting a prismatic bridge truss system to an equivalent, fictitious box girder for evaluation of behavior; 2) to examine the adequacy of the equivalent box girder when the truss system is subject to torsional loads; and 3) to compare the behavior of various shape of truss systems for a sample bridge superstructure.

2 Concept of Equivalent Box Girder

Lateral bracing of an open box section is provided to stiffen the box during shipment and erection and is generally not considered in the design process. However, if such bracing is considered in the design, the dead load stresses applied to an open box section will be supported by a quasi-closed section affording greater torsional stiffness [5]. If the bracing is transformed to an equivalent plate with uniform thickness throughout the length of a rectangular box, then the torsional stiffness can be computed as:

$$I_x = \frac{4A_o^2}{\oint \frac{ds}{t(s)}} = \frac{4(hb)^2}{\left(\frac{2h+b}{t} + \frac{b}{t_{eq}}\right)} \tag{1}$$

rather than as the stiffness of an open section,

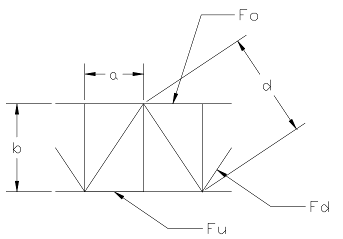
$$I_x = \frac{1}{3}\sum\left(2ht^3 + bt^3\right) \tag{2}$$

where: A_o = area enclosed by the middle lines of the walls.
b = flange width of a rectangular box girder.
h = depth of box girder.
I_x = torsional constant.
s = coordinate measured along the middle line of the walls.
t_{eq} = equivalent plate thickness.
$t(s)$ = t = wall thickness.
$\oint$ = integration around the entire length of the middle line.

The value of the equivalent plate thickness (t_{eq}) is obtained from strain energy or shear rigidity consideration. This thickness is dependent on the configuration of the bracing, the length and the cross-sectional area of the bracing members, and the character of connection. For one of the truss and bracing configurations, the expression for the equivalent plate thickness are given below in equation (3.a) [5, 8].

Equation (3.a) was obtained for both web and flange members from their strain energy considerations assuming all the bracing elements had welded connections which provide fixity and thus moment or bending effects in addition to axial loads. Equation (3.b) was developed only for web members for their shear rigidity considerations.

$$\left\{\begin{array}{ll} t_{eq} = \dfrac{E}{G}\dfrac{ab}{\dfrac{d^3}{K_r F_d} + \dfrac{a^3}{3}\left(\dfrac{1}{F_o} + \dfrac{1}{F_u}\right)} & (3.a) \\ t_{eq} = \dfrac{E}{G}\dfrac{ab}{\dfrac{d^3}{F_d}} & (3.b) \end{array}\right\}$$



where: t_{eq} = equivalent plate thickness, in.
a = vertical member spacing, in.
b = horizontal member spacing, in.
d = diagonal member length, in.
Fd = diagonal member cross-sectional area, in^2.
Fo = upper horizontal member cross-sectional area, in^2.
Fu = lower horizontal member cross-sectional area, in^2.
Fv = vertical member cross-sectional area, in^2.
Ib = vertical member moment of inertia, in^4.
Io = upper horizontal member moment of inertia, in^4.
Iu = lower horizontal member moment of inertia, in^4.
E = modulus of elasticity of steel, ksi.
G = shear modulii of steel, ksi.

$$K_r = (1 + e_x^2 (F/I_y) + e_y (F/I_x))^{-1}$$

e_x = eccentricity of the axial forces referenced to the principal axis, in.
e_y = eccentricity of the axial forces referenced to the principal axis, in.
$F = F_d$ or F_v

In this study, equation (3.a) was used to obtain equivalent plate thicknesses. In these equations, Kr values were assumed to be equal to one, thus leading to less equivalent plate thicknesses than actual, in order to be conservative in calculations.

The conversion of a three dimensional prismatic truss system to an equivalent box girder follows the same concept of converting the lateral bracing of an open box section to an equivalent plate and forming a closed box. Each of the 2-D main trusses and lateral bracing trusses is transformed into an equivalent plate with its thickness computed by equation (3.a).

In the case of an open box section with lateral bracing, the equivalent closed box retains its flanges and webs as components [7]. The stresses and deflections of the braced open box under load can be computed with sufficient accuracy using the equivalent closed box section [6, 9]. In the case of a prismatic truss system, all sides of the prism are converted to equivalent plates, the main components for bending, i.e. the chord members, are retained in the cross-section.

The adequacy of the equivalent box girder is the main topic of this study. In practice truss system designed in 2-D can be converted to a closed box section for which torsional constant, deflection at various points and maximum deflection at the critical section can be easily calculated by the classical mechanics method.

3 Analysis of Structures and Equivalent Box Girders

In order to examine the adequacy of conversion of three dimensional truss systems to equivalent box girders for the evaluation of stresses and deflections, an eighty feet long simple span deck truss bridge was designed with three different cross-sectional shapes for comparison. Twelve 3-D steel truss models and twelve corresponding equivalent steel box girder models were studied. The models were designated t1 to t12 and b1 to b12. The letter "t" represents truss models, whereas the letter "b" represents box girder models.

The truss and box girder models numbered 1 to 4 were rectangular, 5 to 8 were trapezoidal and 9 to 12 were triangular in shape. The rectangular sections, numbered 1 to 4, are 80.52 in. wide and 80 in., 75 in., 70 in. and 65 in. high respectively. The trapezoidal sections, numbered 5 to 8, are 80.52 in. wide on top, 40.26 in. on bottom and 80 in., 75 in., 70 in. and 65 in. high respectively. The triangular sections, numbered 9 to 12, are 80.52 in. wide on top and 80 in., 75 in., 70 in. and 65 in. high respectively. These depths were chosen considering the fact that a typical span to depth ratio of truss bridges is between 18 and 22 [3]. All equivalent steel box girder models were single cell, straight and prismatic.

The dead load on each structure consisted of the self-weight of the structure and the rail weights. The live load consisted of the vehicle weights, the effects of acceleration/deceleration of vehicles and impact, and the wind load acting on the sides along positive y-axis and the ends of vehicles along negative x-axis. Wind load acting on the sides of vehicles and causing torsional force on the structure was studied in five magnitudes. These were the specified design load, an increment of design load by 10% and 20%, and a decrement of design load by 10% and 20% cases.

From common simple span deck truss, lateral bracing and transverse (sway) bracing patterns being widely used in practice, Warren truss was adopted for the vertical main truss; the subdivided system was used for the top and bottom lateral bracing; and cross bracing system was used for the transverse (sway) bracing of the 3-D truss system. The transverse bracing existing in truss models at every 20 ft. were modeled as plate diaphragms in the box girder models. This type of bracing helps to stiffen the box girders and reduce distortion.

All twelve truss models were designed according to AISC-LRFD [2] Provisions for steel members and the maximum deflection was kept less than 1.2 in. (=L/800) as required by AASHTO Specifications [1] to satisfy the desired ride quality of the transportation system. Because the primary objectives of this study are the procedure and adequacy of converting truss systems to their equivalent box girders for examination of torsional behavior, not a direct comparison of 2-D truss versus 3-D truss system, a structural analysis computer program was utilized for the design of the twelve truss models as 3-D frames. For the truss models, AISC rectangular tubes were used as truss members mainly because of their torsional rigidity. In some of the models AISC wide flange shapes were also used as lower chord members in order to acquire the desired bending moment capacities.

Two types of conversion from a 3-D truss system to an equivalent box girder were studied. In the first conversion type which is called the partial conversion, all longitudinal and transverse horizontal members and vertical truss members were kept in the converted box girder while all the diagonals were replaced by equivalent flange, web and diaphragm plates. In the second conversion type which is called the full conversion, only the longitudinal truss chord members were kept in the converted box girder while all other truss members were replaced by equivalent flange, web and diaphragm plates. The partially converted box sections were named as Case 1 - Case 12 and the fully converted box sections were named as Case 1' - Case 12'. The equivalent flange and web thicknesses (calculated by using Eq. 3.a, and etc.).

Five sets of analyses were conducted: finite element analyses of the truss structures and the two types of converted box girders, and the classical analyses of the box girders by structural mechanics. Results from these analyses were to be compared. For the finite element analyses, SAP2000 [11] structural analysis program was used because of its capability of handling all structural components at a time.

All analyses were made in the elastic range, material and geometrical nonlinearities were not considered. The elastic behavior of the structures under service load condition was the main topic of this study. Steel yield stress, modulus of elasticity, shear modulii and poisson's ratio were taken as σ_y = 36 ksi, E = 29,000 ksi, G = 11,200 ksi and υ = 0.3, respectively. The truss structures were analyzed as space frames because of the assumed rigid connection among components.

Since the equivalent box girders are composed of fictitious flanges and webs, the self-weight of these box girders are less than those of corresponding truss systems. Comparison of the truss systems' weights with the partially and fully converted imaginary box girders' weights shows that the truss systems are heavier than the partially converted boxes by 25% and the fully converted boxes by 26% in average. To account for this condition in finite element analysis, self-weights of the equivalent flange and web plates of the converted box girders were ignored while the self-weights of the truss members that were converted to equivalent plates were transferred to the bottom frame members. In the classical method of analyzing the converted box girders by structural mechanics, the vertical displacements of converted box girders were calculated by using Equation (4.1).

$$\Delta_{\max} = 1.2\frac{5wl^4}{384EI_y} + 1.6\sum_{i=1}^{n}\frac{P_i a_i x}{6EI_y l}\left(l^2 - a_i^{\,2} - x^2\right) + \frac{b}{2}\sin\phi_i \tag{4.1}$$

$$\phi_i = \sum_{j=1}^{m}\theta_{i,j} d_j \tag{4.2}$$

$$\theta_{i,j} = \frac{T_{i,j}}{GI_x} \tag{4.3}$$

where: Δ_{max} = maximum vertical displacement, in.

ϕ_i = rotation of the cross-section at point CS in y-z plane, rad.

$\Theta_{i,j}$ = rotation of the cross-section per unit length in y-z plane, 1/in.
w = uniformly distributed load, kips/in.
a_i = distance of concentrated load from the negative end, in.
d_j = distance of torsional load from the negative end, in.
x = distance of critical section from the negative end, in.
b = top flange width, in.
l = span length, in.
I_x = torsional constant, in^4.
I_y = moment of inertia of the cross-section, in^4.
E = modulus of elasticity of steel, ksi.
G = shear modulii of steel, ksi.
P_i = concentrated load, kips.
$T_{i,j}$ = torsional load, kips-in.

In computations n and m refer to number of concentrated load and torque conditions respectively. A two-car vehicle was assumed to move on the 80 ft. long structure. By the classical mechanics method, the maximum vertical displacement of a converted box girder was calculated at point CS under the concentrated load, 566.28 in. from the left end and at the tip of the top flange (b/2 = 80.52/2 = 40.26 in.).

4 Evaluation of Partial and Full Conversion Methods

The computed deflections of the converted box girders are compared for the evaluation of the conversion methods. An eighty inch deep truss system with a rectangular cross section is used as an example. Figure 1 shows the computed maximum vertical deflection of the truss system and the partially and fully converted box girders. Plotted in the figure are the results of five different magnitudes of torsional loads: the code specified and 10% and 20% increase and decrease. The vertical deflections of the partially converted box girder agree quite well with those of the truss system. It appears that retaining the verticals and the horizontal bracing members at the cross diaphragms, the converted box girder provides a better representation of the truss system. Similar results are obtained when the lateral horizontal deflections (in y-direction) are examined. In the direction of the bridge (x-direction), the partially and fully converted boxes have about the same deflections, which agree fairly well with those of the truss system.

For all twelve cases of different bridge cross-sections, the same qualitative results are obtained. The partially converted box girder models have higher torsional rigidity and their vertical and lateral horizontal deflections under code-specified torsion agree better with those computed from the truss systems. Again, in the direction of the vehicle movement, both partially and fully converted boxes have computed deflections in fair agreement with those of the truss systems. Further confirmation of these results can be obtained by examining the deflections of the twelve partially and fully converted box girders under different magnitude of torsional loads. Partially converted boxes always deflect vertically less than the fully converted boxes, and agree better with the trusses.

All computed deflections of the twelve truss systems and their corresponding converted box girders were investigated. Collectively, the deflections in the vertical, lateral and longitudinal (z, y, x) direction of the partially converted boxes are 97%, 90% and 98% of those of the truss systems whereas the corresponding deflections of the fully converted boxes are 133%, 139% and 91% respectively under design load. Obviously, the partial conversion method is preferable. Whereas the equivalent boxes are developed mainly for the evaluation of torsional deflections, it is interesting to compare the computed stresses in these boxes with the corresponding stresses in the truss members. The maximum computed stresses in a "corner member" of the partial and full conversion boxes, and in the corresponding truss chord members, were investigated for the five cases of torsional loads.

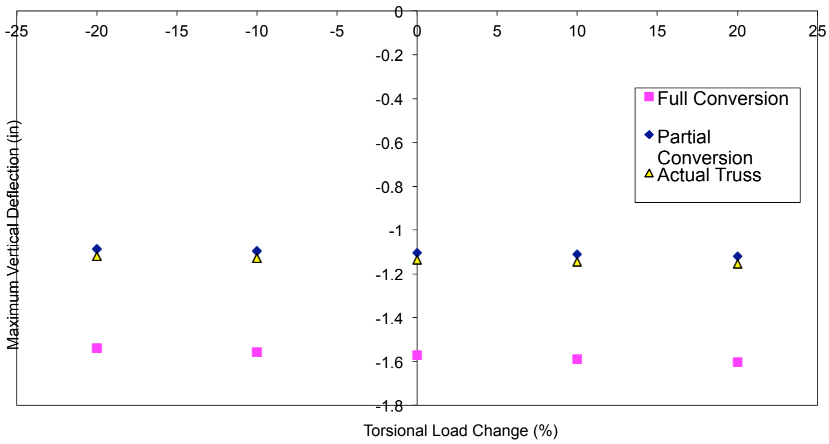


Fig. 1 Maximum Vertical Deflection by Finite Element Analysis versus Torsional Load Change.

For all twelve trusses systems and their equivalent box girders, the bending stresses are always lower in the corner members of the boxes than in the truss chord members. For example, for the 80 in. deep rectangular cross section under design load (Case 1 in Table 1) the bending stress in the truss chord is 19.56 ksi while those in the partial and full conversion boxes are 15.10 ksi and 16.53 ksi, respectively. The partial conversion box, being better than the full conversion box in examination of deflection, is less accurate in computed stresses when the stresses are compared to those in the truss members. For the four rectangular truss systems under design loads (Cases 1 to 4 in Table 1) the average ratio of bending, horizontal shear and vertical shear stresses in the corner member of the partial and full conversion boxes are 0.79, 0.85; 0.88, 0.92 and 0.89, 0.85, respectively.

It must be recognized that the equivalent box girders are quite different from the trusses in distribution of forces and stresses among their component members. The computed stresses in the corner members served as an indication that the converted box girders are reasonable and adequate tools for the examination of deflections. Based on the comparison of deflections, the partial conversion box girder model is recommended for the procedure.

One of the primary objectives of this study is to examine the possibility and accuracy of using an equivalent box girder for evaluating deflections of truss bridges, which have the trusses designed as two dimensional members. The equation for computation of vertical deflection of top flange tips of box girders by the classical method has been given earlier as Eq. 4.1. The computed deflections of the twelve equivalent boxes, by both partial and full conversion method, were investigated for the five torsional load cases.

	Bending Stresses			Shear Stresses					
Case	bx	b'x	tx	by	b'y	ty	bz	b'z	tz
1	15.10	16.53	19.56	-1.45	-1.52	-1.61	-0.29	-0.29	-0.33
2	13.82	15.12	17.35	-1.21	-1.26	-1.35	-0.23	-0.23	-0.27
3	13.77	14.74	17.31	-1.18	-1.24	-1.36	-0.24	-0.22	-0.27
4	13.70	14.32	17.23	-1.17	-1.22	-1.38	-0.24	-0.22	-0.27
Average Stress (ksi)	14.10	15.18	17.86	-1.25	-1.31	-1.43	-0.25	-0.24	-0.28
Average Stress Ratio	0.79	0.85	1.00	0.88	0.92	1.00	**0.89**	**0.85**	1.00
5	13.29	14.70	16.43	-1.14	-1.24	-1.24	-0.36	-0.22	-0.34
6	13.12	14.14	16.28	-1.15	-1.20	-1.27	-0.43	-0.22	-0.35
7	13.42	14.27	16.54	-1.13	-1.20	-1.26	-0.39	-0.22	-0.37
8	13.17	13.93	16.19	-1.14	-1.18	-1.30	-0.41	-0.22	-0.38
Average Stress (ksi)	13.25	14.26	16.36	-1.14	-1.21	-1.27	-0.40	-0.22	-0.36
Average Stress Ratio	0.81	0.87	1.00	0.90	**0.95**	1.00	**1.10**	0.61	1.00
9	12.85	14.23	15.62	-1.13	-1.08	-1.23	-0.62	-0.23	-0.37
10	12.95	14.24	15.63	-1.14	-1.08	-1.23	-0.70	-0.24	-0.39
11	12.98	14.26	15.56	-1.14	-1.07	-1.24	-0.72	-0.24	-0.40
12	12.84	13.94	15.25	-1.13	-1.07	-1.24	-0.74	-0.25	-0.39
Average Stress (ksi)	12.90	14.17	15.51	-1.14	-1.07	-1.23	-0.69	-0.24	-0.39
Average Stress Ratio	**0.83**	**0.91**	1.00	**0.92**	0.87	1.00	1.79	0.62	1.00
Average Stress (ksi)	13.42	14.53	16.58	-1.18	-1.20	-1.31	-0.45	-0.23	-0.34
Average Stress Ratio	0.81	**0.88**	1.00	0.90	**0.91**	1.00	**1.30**	**0.68**	1.00

Note: prefix b and b' for partially and fully converted box girders, t for truss systems.

Table 1 Bending and Shear Stresses by Finite Element Analysis under design load.

The computed maximum vertical deflection at the tip of the equivalent box girder for the 80 in. deep rectangular cross section were compared with those computed deflections of the truss. The equivalent boxes from partial and full conversion have less vertical deflection than the truss for all cases of torsional loads. Since the partial conversion method is considered to be providing a better equivalent box girder model for the evaluation of deflection of trusses by the finite element analysis, a comparison of deflection of the partial conversion box by both the finite element analysis and the classical method is made. The computed deflections for the 80 in. deep rectangular cross section are presented for the five torsional load cases. In every case, the finite element analysis provides accurate computed values in comparison to truss deflections. The classical method, on the other hand, always underestimates the deflection by a fairly significant amount.

Table 1 shows that, under design loads, the average computed flange tip vertical deflection of the partial conversion boxes is 71% of that of the trusses. It is 73% for the full conversion boxes. The cause of this relatively large difference could be many. The most significant probably is the basic difference between the finite element analysis and the classical method of computing the deflection.

While the finite element analysis considers all local deflections and deformation of the components and the structure, including that of the local deflection of the fictitious flange and web plates of the equivalent boxes, the classical structural mechanics procedure of torsion and bending analyses adopted for the study assumes that the box girders retain their cross-sectional shapes. It appears that, by adding a multiplication factor of (1/0.71), or about 1.4; the deflection of the three dimensional truss systems can be estimated by using the fictitious partial conversion equivalent box girders.

5 Discussion on Cross Sectional Shapes

The utilization of rectangular, trapezoidal and triangular cross sections of a truss bridge to examine the adequacy of equivalent box girders for deflection control provides a chance for comparing these cross sectional shapes. Traditionally, deck bridge truss systems consist of two vertical main trusses and the top and bottom level lateral bracing members forming a rectangular cross section. Seldom are inclined main trusses used for deck bridges. From the results of this study, it appears that there is no reason to exclude the trapezoidal or the triangular cross section if all other conditions are about equal.

The twelve truss systems of this study, with 80 ft. span and specified depth of cross section, were designed to satisfy the deflection limitation of L/800 which is 1.20 in. For the cross sectional depth of 80 in., the weight of the rectangular, trapezoidal and triangular cross sections are about the same with the chosen component member sizes. The partial conversion equivalent box girder sections all have about the same magnitude of vertical deflection, being 96%, 98% and 98% by the finite element analysis for the rectangular, trapezoidal and triangular cross-section, respectively, when compared with those of the corresponding trusses (Table 1). The corresponding values are 73%, 70% and 70% when the classical method is used for deflection calculation. All these indicate that either the trapezoidal or the triangular shape can be utilized if desired.

Examination of the bending stresses in the chord members of the truss systems also leads to the same conclusion stated above. The bending stresses in the chord members of the rectangular, trapezoidal and triangular truss systems are 17.86 ksi., 16.86 ksi. and 15.51 ksi. respectively (Table 1). The ratio of computed partial conversion box girder corner members are 79%, 81% and 83%. All are practically equal. Whether a trapezoidal cross-section is more practical or a triangular cross section is more pleasing aesthetically, is to be judged by the circumstances of the bridge. The results of this study suggest that both can be utilized, as does the rectangular cross sectional shape.

6 Conclusions

Objectives, methodology and results of this study are summarized below:

1. Twelve deck truss systems of 80 ft. span length and with different cross sectional depth and shapes were designed by limitation of deflection (L/800)
2. These truss systems were converted to equivalent box girders by using the equivalent thickness concept, which was developed for lateral bracing systems of open box section. Each of the two dimensional trusses and the lateral bracing between them was converted to an equivalent plate.
3. Two methods of conversion were examined. The partial conversion model retains in the equivalent box section the verticals of the main trusses and the transverse horizontal members of the lateral bracing system, as well as the chord members of the main trusses. In the full conversion method, only the chord members of the main trusses are retained in the equivalent box girder.
4. The deflections of the truss systems and of the converted equivalent box girders were both computed by finite element analysis. Results indicate that partially converted boxes provide a better representation for truss system deflections.
5. The deflections at points of the equivalent box girders were also computed using the classical method of structural mechanics.
6. Results of deflections computed from partially converted equivalent boxes by the classical method are lower than those corresponding deflections of truss systems, computed by the finite element analysis. The ratio is about 0.7/1.0 (or 1.0/1.4). A multiplication factor of 1.4 is suggested.
7. From the results of corresponding deflections, stresses and structure weight of rectangular, trapezoidal and triangular cross-sections, all three shapes can be adopted adequately for deck truss bridges.
8. A more exhaustive examination of the deflections of truss systems and their equivalent box girders is recommended. Various span lengths, cross sectional shapes and different loads need to be considered. The adequacy of the multiplication factor needs to be further investigated.
9. Equivalent model that is developed by available analytical solutions, can provide guidance for quick preliminary study and proportion the members, such as guiding the layout and amount of bracing.

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